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BANKRUPTCY PREDICTION APPLYING MULTIVARIATE TECHNIQUES

PREDIKCIA BANKROTU S VYUŽITÍM VIACROZMERNÝCH TECHNÍK

Abstract: *The paper focuses on the analysis of the corporate bankruptcy prediction using selected statistical multidimensional methods. Existing multidimensional methods are a suitable tool for predicting the bankruptcy of companies, for their graphical representation in space, the identification of clusters of companies with the same bankruptcy preconditions, as well as the identification of bankruptcy factors. The research was carried out on a sample of 343 heat management companies in Slovakia. All of these companies operate local district heating systems. Within this group, there are companies that have a monopoly position in a given geographical area. Of the multidimensional methods, the Principal Component Analysis (PCA) method and the Multidimensional Scaling (MDS) method were used. The resulting graphical representation of both methods yielded significant results. The paper identified the main factors in predicting bankruptcy. It has been found that it is possible to predict bankruptcy of the analyzed sample of companies using three main factors that capture 70% of the information from the applied indicators. It follows that it is not necessary to apply a large number of indicators to reveal the financial situation of companies. In addition, similar characteristics of enterprises make it easier to predict bankruptcy in larger samples.*

Keywords*: Bankruptcy, Multidimensional Scaling, Prediction, Principal Component Analysis.*

Kľúčové slová*: Bankrot, multidimenzionálne škálovanie, predikcia, analýza hlavných komponentov.*

JEL classification: M20, G33, C53

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Introduction

Prediction of business financial distress and bankruptcy is a subject that has gained great interest from researchers in the field of finance. There are a number of methods designed to assess businesses` financial health and predict their possible bankruptcy. Several of these methods are based on mathematical and statistical methods, most of which are regression models or discriminant analysis models. Researchers who deal with this issue divide the mentioned methods into different groups. Araghi and Makvandi [1] classify prediction models to statistical models, models based on artificial intelligence and theoretical models. Balcaen and Ooghe [2] divide failure prediction models into classic statistical models and alternative methods. In the following text, we present the methods that these researchers include in mentioned groups and the example of the author who applied them. Classic statistical models include univariate analysis [3], risk index models [4], multiple discriminant analysis [5], conditional probability models – Logit analysis [6], probit analysis [7], linear probability modelling [8]. As alternative methods, they mention multi-logit analysis [9], survival analysis [10], dynamic event history analysis [11], multidimensional scaling [12]. decision trees [13], expert systems and neural networks [14]. Alternative methods may include also other multidimensional techniques in addition to the above-mentioned, namely cluster analysis, factor analysis, principal component analysis or correspondence analysis. In this paper we applied multidimensional scaling (MDS) and principal component analysis (PCA).

Multidimensional techniques allow a graphical representation of the financial position of companies, while creating groups of companies with the same characteristics and the same financial difficulties. They make it possible to reduce the number of evaluated financial indicators and to reveal indicators that are essential for the identification of bankruptcy [15].

The aim of the paper was to minimize the number of variables that can be used to predict the bankruptcy of businesses, to detect the symptoms of bankruptcy and to identify clusters of businesses within which there are businesses with similar values of the given variables.

The originality of the research lies in the application of multidimensional techniques to identify symptoms of a possible bankruptcy of businesses.

The remainder of the paper is structured as follows. Section 2 is literature review. This section defines the term bankruptcy and provides the overview of theoretical knowledge about MDS and PCA. The third section `Methodology` describes the data, the analysed sample of businesses and theoretical background of applied methods. Section 4 includes results and discussion of the results achieved. This section lists and compares the results of bankruptcy prediction applying MDS and PCA. Section `Conclusion` summarizes the essential conclusions resulting from the research and brings significant findings.

Literature review

Bankruptcy prediction is one of the crucial issues which have been often studied in accounting and finance literature [16]. From a methodological point of view, the prediction of bankruptcy is a binary classification problem which aim is to differentiate between solvent and insolvent groups of businesses in the best way [17] In: [18].

Altman and Hotchkiss [19] specify two types of bankruptcy. One type is insolvency in a bankruptcy sense, which usually indicates a chronic rather than a temporary condition. A business finds itself in this situation when its total liabilities exceed a fair valuation of its total assets A second type of bankruptcy is company's formal declaration of bankruptcy in a federal district court, along with a proposal to liquidate its assets or attempt recovery.

Models for bankruptcy prediction among others include multivariate techniques PCA and MDA.

PCA is one of the most widely used multivariate techniques in statistics [20]. Preisendorfer and Mobley [21] In: [22] states that the origins of this statistical technique are linked to Singular Value Decomposition (SVD), independently derived by Beltrami [23] and Jordan [24] in the form that is directly related to PCA. The foundations of PCA were laid by Pearson [25], the general procedure of this technique as we know it today was given by Hotelling [26]. According to Jolliffe [22] the motivation of Hotelling was that there may be a smaller basic set of independent variables which determine the values of the original p variables. These variables are called factors in psychological literature, but to avoid confusion with other uses of the word factor in mathematics, Hotelling introduced the alternative term components. He suggested to choose these components so as to maximize their successive contribution to the sum of the variances of the original variables and called the components derived in this way principal components. The analysis which finds such components was then called method of principal components.

According to Jackson [27] the development of PCA technique has been quite uneven in the following years. There was a great deal of activity in the late 1930s and early 1940s. Things then subsided for a while until computers were designed, which allowed these techniques to be applied to problems of appropriate size.

PCA method is attractive mainly because the main components are uncorrelated. Instead of investigating a large number of original variables with complex internal links, the user analyses only a small number of uncorrelated main components. Principal component analysis is also part of exploratory data analysis. Reduction of the data dimensionality is often used in the construction of comprehensive indicators as linear combinations of original variables. The use of the first main component as a comprehensive indicator is common in the field of economics, sociology and medicine. The first two or three main components are used primarily as techniques for displaying multidimensional data. In many cases, PCA is only one part of a more complex analysis [28] In: [29]. This method was used in such a way by Succurro et al. [30] who implemented tandem analysis based on the use of PCA and Logit model or Canbas et al. [31] who created an integrated early warning system for predicting banks failure applying discriminant analysis, PCA, logit and probit analysis.

The second multivariate technique applied in this research is MDS. It is a "statistical method that optimally maps proximity data on pairs of objects (i.e. data expressing the similarity or the dissimilarity of pairs of objects) into distances between points in a multidimensional space (usually 2 or 3 dimensions)" [32]. Objects can be people, attributes, stimuli, countries, etc., measurements can be correlations between test items, similarity of politicians, dissimilarity of mobile phones, etc. The main aim is to represent these objects as points in low-dimensional (usually 2-dimensional) space in such a way that the distances among the points represent the (dis)similarities as good as possible. The motive for this is the visualization of the data in a picture which makes the data structure much more accessible to researchers than a data matrix with many numbers [33].

According to Neophytou and Molinero [34] In: [18] MDS visualizes the hidden relationships between data and reduces them into multidimensional coordinates. The applicability of MDS is broad and this method can be potentially used across many disciplines such as psychology, psychophysics, neuroscience, marketing, political science, sociology, ecology and others [35].

The first algebraic approach to MDS is classical MDS, which assumes metric data as inputs. This approach has been independently proposed by Torgerson [36], Gower [37], and Kloek and Theil [38]. Gower [37] In: [33] was the first to realize that the reduction of principal component analysis dimensions has a dual method that can be obtained by performing classical MDS on Euclidean distances of data matrix rows.

Another basic approach to MDS is ordinal (also called non-metric) MDS. This approach is used in situations where one value is not enough to capture the fact. Non-metric MDS uses the order of the distances between the objects, not their actual values. The pioneer of this approach was Shepard [39] [40], followed by Kruskal [41] [42], who suggested the loss function called Stress, and [43].

Before the application of MDS in bankruptcy prediction, this method was used in accounting and finance. An early application of MDS in accounting was reported by Green and Maheshwari [44]. Subsequently Frank [45] compared international accounting principles using MDS, Libby [46] and Bailey et al. [47] applied the MDS to clarify audit issues, [48] used the MDS to study the value of accounting information to investors In: [34].

Multidimensional scaling has been used as the alternative model for the analysis of business failure because it bypasses many of the shortcomings of discriminant analysis and Logit model. First authors who applied this method for business failure prediction were Mar-Molinero and Ezzamel [12]. In 2001 Mar-Molinero and Serrano-Cinca [49] extended this work and suggested a way in which MDS can be used as an alternative to discriminant analysis or Logit model in order to classify companies as failed or continuing.

The MDS algorithm does not make any assumptions about the distribution of financial indicators on which the analysis is performed. MDS has an important benefit: it visualizes the main features of the situation and thus allows the incorporation of non-quantitative information into the analysis. The reasons why a particular company fails or does not fail and the risk of failure of a particular company are assessed and also visualized. In this way, the MDS opens the door for the judgment to supplement statistical analysis [34].

Material and Methods

The sample of businesses for carrying out this research consisted of 343 companies doing a business in the field of heat supply. The data from financial statements of companies for the year 2016 were obtained from CRIF - Slovak Credit Bureau, s.r.o. [50]. According to SK NACE Rev. 2 the sample of businesses falls under section D "Supply of electricity, gas, steam and cold air". Regarding the legal status of businesses, 15% of them are joint stock companies and the remaining 85% are limited liability companies. The results of the financial analysis show that the analysed companies can have a liquidity problem. Despite the fact that the average value of the Current ratio is 3.92, median is 0.951. Value of this indicator lower than 1 means higher financial risk. This is also reflected in the negative value of net working capital. The analysed companies achieve high creditors payment period, which results in a negative value of cash-to-cash. The assets of these companies change on average once a year. The average return on assets is 5%. The capital structure of these companies is 35:65 in favour of equity. The performance of companies assessed by the EVA indicator indicates that businesses from the analysed sample can be threatened with bankruptcy.

When applying PCA and MDS we used 9 financial indicators, 8 of them were the same indicators which were used by Premachandra et al. [51]. We applied these indicators: TDTA – total debt / total assets used as a leverage measure which indicates long-term financial obligation, CLTA – current liabilities / total assets which indicates a lack of cash flow to fund business operations, CFTA – cash flow / total assets, NITA – net income / total assets, WCTA – working capital / total assets, CATA – current assets / total assets, EBTA – earnings before interest and taxes / total assets, EBIE – earnings before interest and taxes / interest expense. Due to the lack of the data necessary for the calculation, we replaced the last Premachandra`s indicator by similar one ETD – equity / total debt, which was used by Altman [52].

The relationship between MDS and PCA has been studied by many researchers. According to Mar-Molinero and Serrano Cinca [49] who takes into account research of Lingoes [53], Shepard [54], MacCallum [55] and Balloun and Oumlil [56], the general conclusion is that both non-metric PCA and MDS yield the same message about the data. Hout et al. [35] also states that PCA achieves similar results to MDS, but he further discusses this idea. According to this author the PCA approach is mathematically identical to the metric MDS based on Euclidean distance. By comparison, non-metric MDS is better able to maintain point-to-point distances in the final configuration. In essence, the difference can be best described in terms of the research objectives: PCA focuses more on the dimensions themselves and fitting the variance as closely as possible, while MDS focuses more on the relationships between scaled objects.

In this paper we used MDS and PCA methods to explore relationships between financial ratios of analysed businesses and the differences between bankrupt and non-bankrupt businesses from the analysed sample.

The aim of MDS is to find dimensions that will allow to explain the identified similarities or differences between objects. Within the MDS, any kind of similarity or distance can be analyzed based on the so-called proximity matrix. Unlike other multidimensional methods, MDS does not require a precise definition of the variables used when comparing objects.

The proximity matrix contains three different types of data, namely the distances between the objects d_{ij}, the similarities between the objects S_{ij} and the values of the variables (columns) for the individual objects (rows) x_{ij}. The distance (dissimilarity) d_{ij} represents the distance between objects. The distance matrix D is symmetric [57].

The distance between points i and j is calculated using the Euclidean distances of the objects according to formula *(1):*

$$d_{ij} = \sqrt{\sum_{k=1}^{p}\left(x_{ik} - x_{jk}\right)^2} \qquad (1)$$

where p is the number of dimensions, x_{ik} *is the value of the data from the row i and the column k [58].*

The similarity of S_{ij}, expresses how close two objects are. The degree of similarity is calculated for each pair of objects. The similarity matrix S is again symmetric

The similarity of objects can be converted to a distance according to the relationship *(2):*

$$d_{ij} = \sqrt{S_{ii} + S_{jj} - 2S_{ij}} \qquad (2)$$

where d_{ij} *represents the distance i and j of the object,* S_{ij} *expresses the similarity of objects,* x_{ij} *are the values of variables, from which the correlation matrix of objects* R *is calculated first and then the matrix of Euclidean distances of objects* D *is calculated, too [57].*

How well the multidimensional object scaling model fits the given data can be assessed by a measure of goodness of fit using the statistical measure *Stress*. The most widely used formulation of the measure of goodness of fit in this respect is the Kruskal's *Stress* [41], which is calculated according to formula *(3)*:

$$Stress = \sqrt{\frac{\sum_{k=1}^{m}\left(d_{ij} - \hat{d}_{ij}\right)^2}{\sum_{k=1}^{m} d_{ij}^2}} \qquad (3)$$

where $\hat{d}_{ij}$ *expresses the predicted distance between objects i and j and* d_{ij} *is the actual distance between objects i and j.*

If the value of the *Stress* criterion is close to zero, the fit of the objects using multidimensional scaling reaches the best values. In general, the smaller the value of the *Stress* criterion, the more the calculated and entered object coordinates fit. According to Kruskal [41] *Stress* around 0.20 means insufficient overlap, 0.10 sufficient, 0.05 good, 0.025 excellent and 0.00 perfect fit.

An important task is to determine the total number of required coordinates in the MDS model. Each coordinate represents a latent variable. The goal of MDS is to keep the number of coordinates as small as possible (usually we choose 2-dimensional, maximum 3-dimensional space). If the outcome is a higher number of coordinates, the multidimensional scaling technique is not suitable for the analysis of the data. The number of coordinates is chosen based on the lowest possible value of the Stress criterion.

As already mentioned, the output of multidimensional scaling is the so-called multidimensional object map that allows for comparison of the positions of the examined objects and dimensions. A multidimensional map of objects is to be found in the table and figure below. The graphical form of a multidimensional map makes it possible to explain the input data matrix (proximity matrix) usually using a two-dimensional scatter plot. A multidimensional map of objects does not strictly lean towards one point. Similar objects are close to each other, different are farther apart. If the map is created by the metric method, the distances in the graph are very similar to the distances calculated in the table. In the case of non-metric output, only the order of individual objects is preserved

A particularly interesting characteristic of MDS maps is robustness to discordant observations. If the distance between a point and the rest is very large, this point will be located far from the others. The proximity relationships between other points will not be affected (although care must be taken when using interpretative techniques such as profit analysis). This is in contrast to other techniques used to analyse failure, which tend to be sensitive to outliers, such as Data Envelopment Analysis (DEA) [49].

The aim of PCA is to reduce the dimensionality of a dataset, while preserving as much variability as possible. This method can be based on either the covariance matrix and the correlation matrix [59]. It is a multivariate technique in which a number of related variables (X_1, X_2 …, X_k) are transformed to a set of uncorrelated variables – principal components (PC_1, PC_2, …, PC_k) [60]. The number of principal components is less than or equal to the number of original variables. These components are synthetic variables of maximum variance, calculated as a linear combination of the original variables. The first principal component represents as much variability in the data as possible, and each succeeding component represents as much of the remaining variability as possible [61]. Formally *PCj* can be written according to formula *(4)* [60]:

$$PC_j = a_{1j}X_1 + a_{2j}X_2 + \cdots + a_{kj}X_k$$
(4)

where a_{ij} *– component weights, j = 1, 2, ..., q.*

If the data is concentrated in a linear subspace, this provides a way to compress the data without losing a large amount of information and simplifying the representation. By choosing eigenvectors with the largest eigenvalues, we lose as little information as possible in the mean-square sense. The PCA therefore offers a comfortable way to check the trade-off between losing information and reducing the dimension of the initial data representation [61].

It is common to use some predefined percentage of the total variance explained to decide how many principal components should be retained (70% of the total variability is commonly used, if subjective, cut-off point), although the requirements of graphical representation often result in the use of only the first two or three principal components. Even in these cases, the percentage of the total variance is the basic tool for evaluating the quality of these low-dimensional graphical representations of the data set. The emphasis in PCA is almost always on the first few principal components, but there are circumstances in which the last few components may be of interest, such as in detection outliers or in some applications of image analysis [59].

Results and Discussion

Using the PCA method, it was possible to identify the main factors of the impeding bankruptcy of companies. Based on the rules for determining the number of main factors, we identified 3 main principal components. These 3 principal components explain 70% of the total variance. These components were derived as eigenvalues from an already created correlation matrix. With each eigenvalue, it was possible to describe the part of the total variability of the original variables, which is expressed in %. The percentages are shown in the Figure 1.

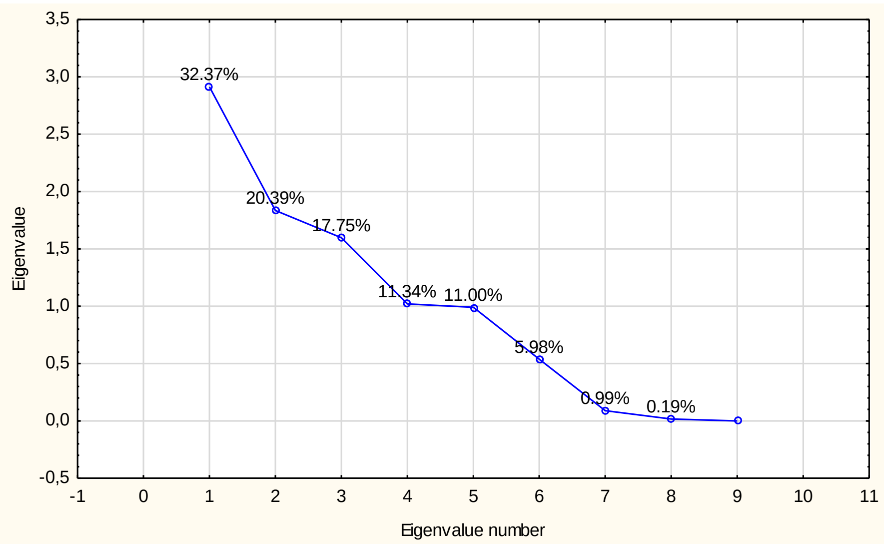

Fig. 1: Eigenvalues of correlation matrix
Source: authors, processed in software Statistica

Table 1 shows the correlation coefficients of the variables with the principal components. The correlation coefficient expresses the extent to which the original variable affects the new principal component, i.e. the higher the coefficient, the more the original variable affects the new principal component.

Table 1 shows that principal component 1 is strongly inversely related to the variable CFTA, NITA and EBTA. These variables describe the profitability of the company.

There is a strong directly proportional relationship between the principal component 2 and the variable CATA. This principal component also has a strong inverse relationship to the TDTA variable. Based on this relationship, it can be stated that the principal component 2 describes the structure of assets, indebtedness and liquidity of the company.

Principal component 3 shows a strong inversely related relationship to the WCTA variable. Based on the above, it can be stated that this factor captures information on the liquidity of the company. If we included the principal component 4 in our model, we could have got information on the structure of capital/ indebtedness in the form of the ETD variable.

Tab. 1: Relationships between variables and principal components.

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8
	Factor coordinates of the variables, based on correlations							
CFTA	-0.965998	0.026104	0.082966	0.004690	0.001083	-0.007916	0.242814	-0.015473
NITA	-0.978147	0.157507	0.008291	-0.005939	0.008342	-0.033887	-0.087096	0.097528
WCTA	-0.062434	0.025869	-0.962078	-0.259380	0.042861	-0.024779	0.010233	-0.001988
CATA	0.152874	0.817398	-0.361376	-0.293974	0.093396	-0.287018	0.019060	-0.003238
EBTA	-0.977305	0.112700	0.037583	-0.020925	0.008409	-0.034538	-0.148911	-0.082980
EBIE	0.003698	0.138357	-0.084992	0.050839	-0.984202	-0.048779	0.001191	-0.000601
ETD	0.001917	0.056962	-0.347097	0.891981	0.092253	-0.268556	0.002857	-0.002077
TDTA	-0.028017	-0.768617	0.133149	-0.260343	-0.026898	-0.567620	-0.003667	0.002608
CLTA	0.200641	0.717320	0.622373	-0.010835	0.042523	-0.236366	0.007210	-0.000978

Source: authors, processed in software Statistica

Based on the above, it can be stated that the variables that can be used to identify the probability of bankruptcy of the company are indicators of profitability, liquidity and indebtedness.

In order to graphically represent the above relationships between the principal components and variables, a figure of variable projection (Figure 2) was drawn up. This Figure shows the original variables in the new coordinate system of principal components 1 and 2. The influence of variables on principal components is evaluated by comparing the vectors of individual variables. The longer the vector, the stronger the effect of the variable; the smaller the angle between the vector and the respective principal component axis, the stronger the influence of the variable on the given component.

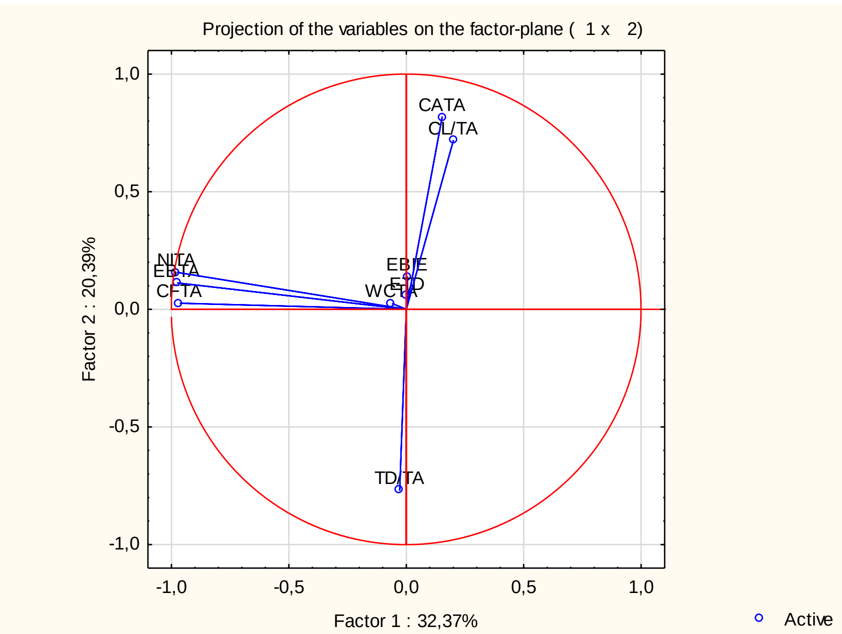

Fig. 2: Projection of variables

Source: authors, processed in software Statistica

From the graphical representation of the projection of variables, it is clear that there is a strong inversely proportional relationship between the principal component 1 and profitability indicators. The principal component 2 has a strong directly proportional relationship with the variables CATA and CLTA and an indirectly proportional relationship with the variable TDTA.

Projection of cases (figure 3), which was processed using the PCA method, enables us to show individual companies in a two-dimensional space, while each company and the observations related to it are given by the values of all applied variables simultaneously. This case projection, processed using the PCA method, suggests that the whole analysed sample of companies creates a significant cluster in the space around the beginning of the coordinate system. This cluster is given by the coordinates (x: 5,-5; y: -4,4). Outside this cluster are enterprises that achieve extreme values of variables. The space for cluster analysis was defined by the principal components 1 and 2. The principle component 1 shows information on EBTA, NITA and CFTA indicators, while these variables are inversely related to the principal component 1. The principal component 2 shows information on the share of current assets in total assets and the share of short-term liabilities in assets. Therefore, we can say that principal component 2 informs us about the resources that can be used for the development of the company and also about the liquidity of the company.

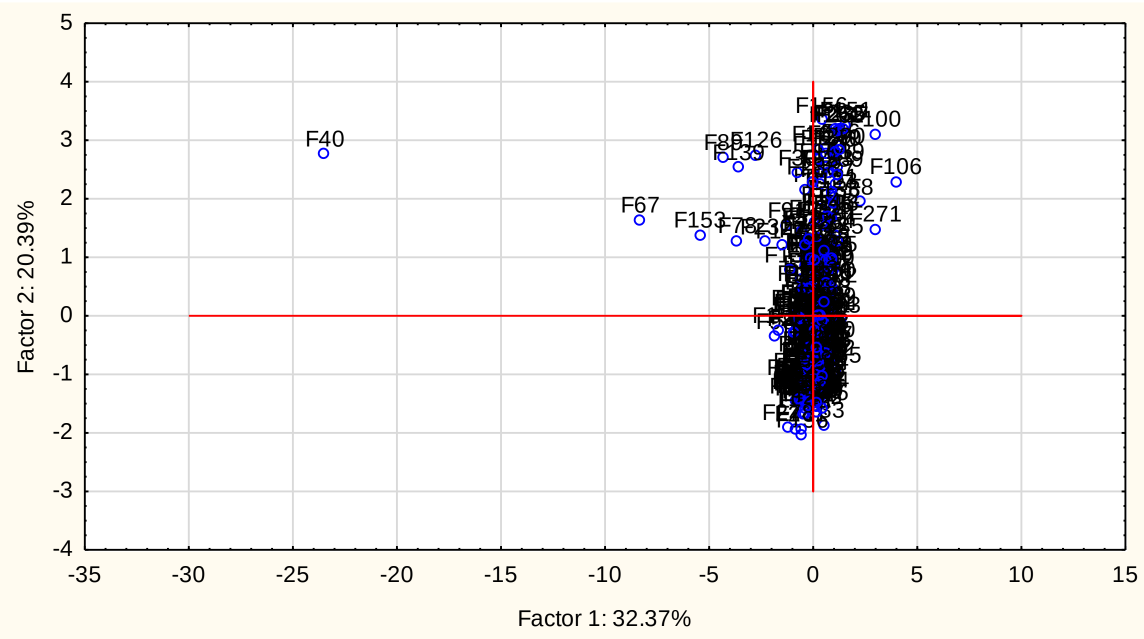

Fig. 3: Projection of cases
Source: authors, processed in software Statistica

For a more detailed description of the companies in the individual quadrants of the case projection, we prepared a larger version.

In quadrant A of the case projection (figure 4), there are companies that perform worse in terms of profitability and achieve better results in terms of liquidity. In order to improve the company's results in the future and prevent bankruptcy, it is necessary to pay attention to improving their profitability.

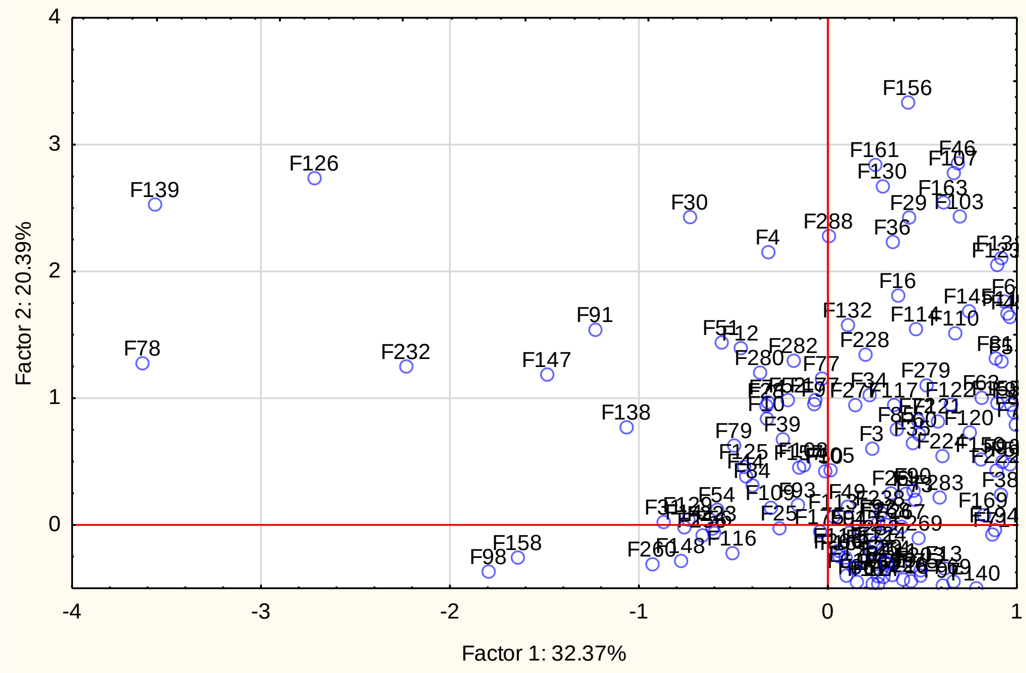

Fig. 4: Quadrant A of the projection of cases
Source: authors, processed in software Statistica

In quadrant B of the case projection figure (quadrant on the top right – figure 5) are companies that achieve very good liquidity results and good profitability results. These companies are among the best performing companies which do not have to worry about going bankrupt.

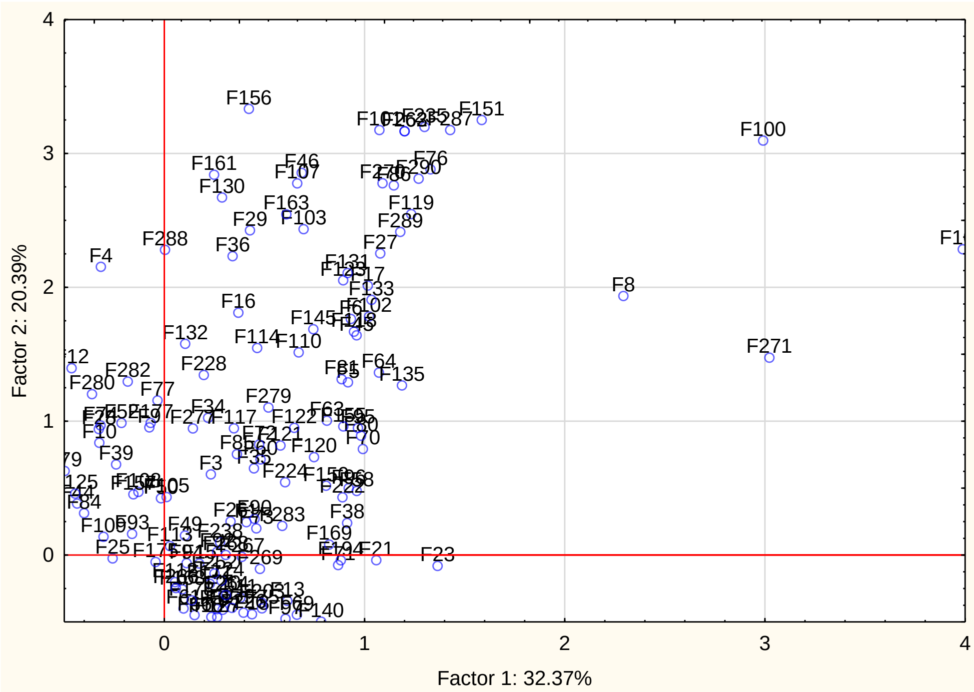

Fig. 5: Quadrant B of the projection of cases
Source: authors, processed in software Statistica

Very good results in terms of profitability and liquidity problems are shown by companies located in quadrant C of the case projection figure (quadrant at the bottom right) (figure 6).

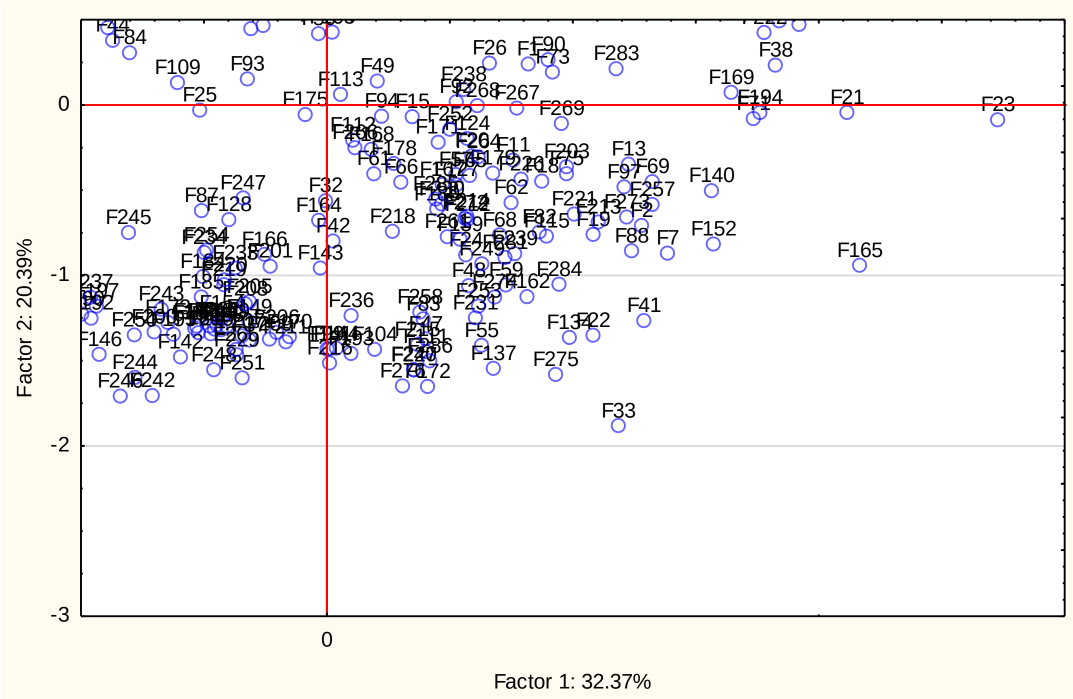

Fig. 6: Quadrant C of the projection of cases
Source: authors, processed in software Statistica

Businesses located in quadrant D of the case projection figure (bottom left quadrant) (figure 7) have problems with the principal component 1 and 2 and the variables that describe these components.

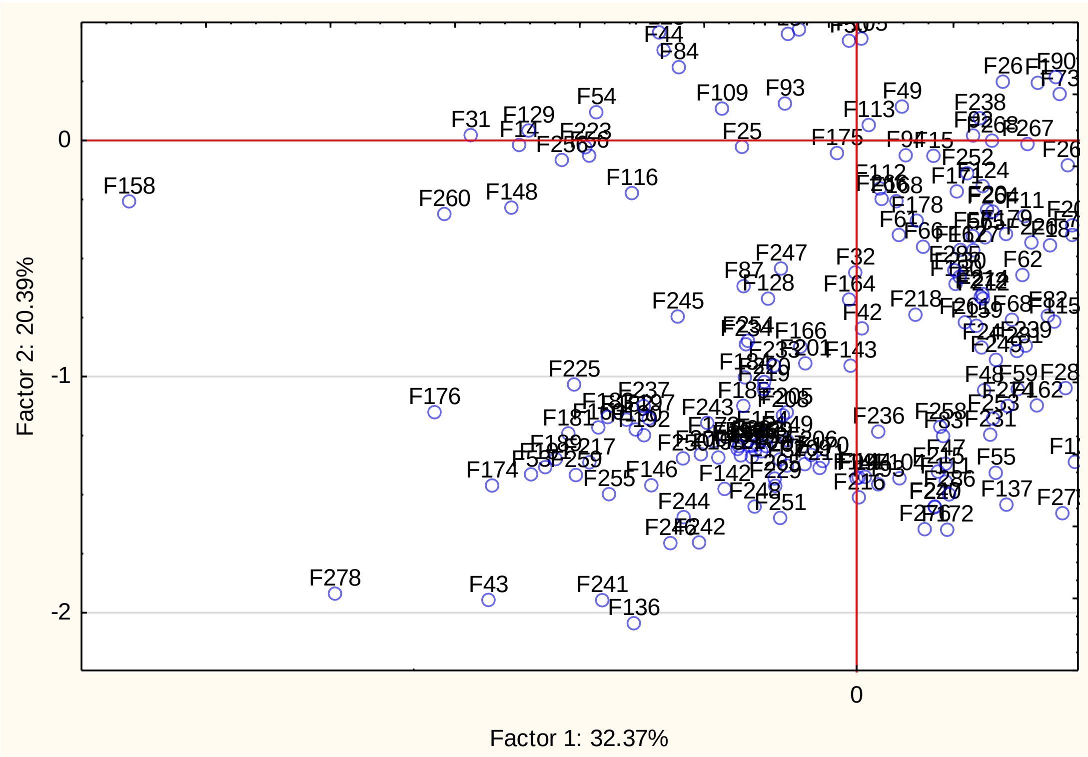

Fig. 7: Quadrant D of the projection of cases
Source: authors, processed in software Statistica

These companies achieve the worst results in the given variables and it is assumed that they are in financial distress, thus meeting the bankruptcy criteria. This argument is reinforced by the fact that the principal component 2 is inversely proportional to the variable indebtedness of the company. Therefore, this quadrant features companies that show problems in the area of corporate debt.

In addition to the PCA method, the MDS method was also used. This subjective map created the preconditions for determining similarities between companies on the basis of used variables. The measure of goodness of fit was Kruskal's criterion of maximum likelihood - Stress and Shepard diagram.

The Figure 8 shows that companies tend to form clusters. It also allows us to identify relationships that make it easier to identify clusters and the structure of all objects.

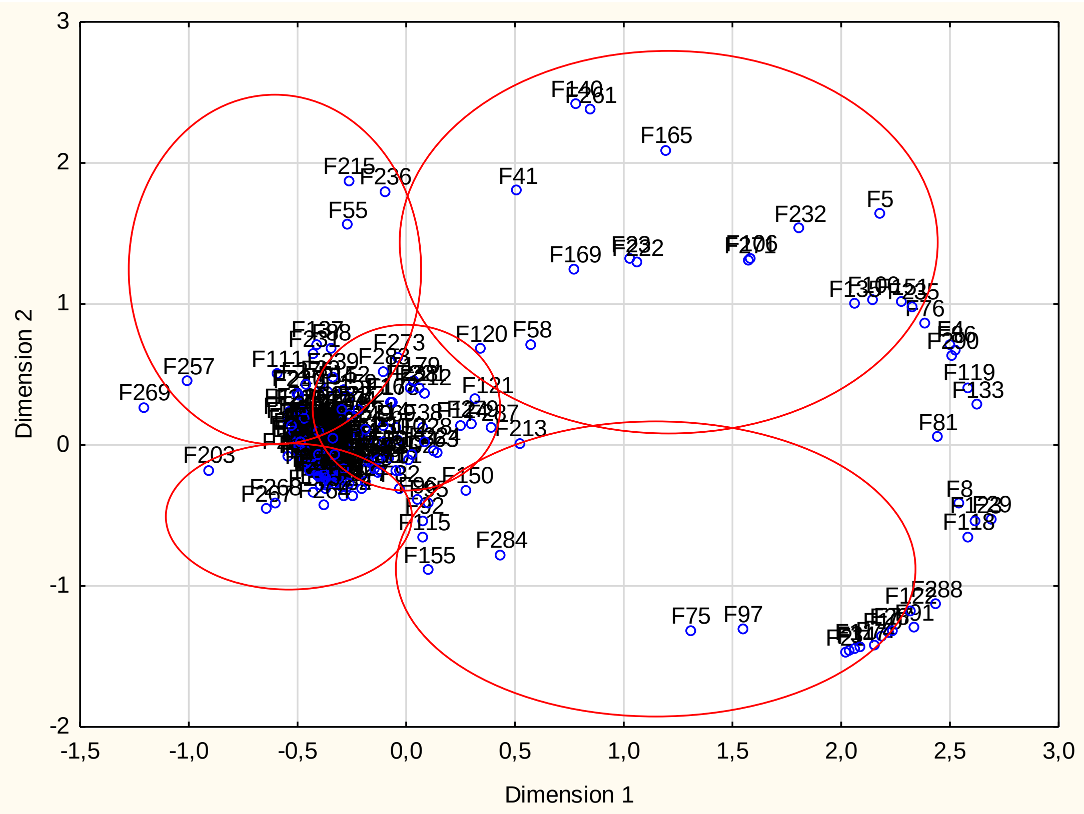

Fig. 8: MDS map

Source: authors, processed in software Statistica

The Shepard diagram (figure 9) shows the calculated distances depending on the actual similarities. All points lying close to the curve represent a good model. Points far from the curve represent insufficient fitting.

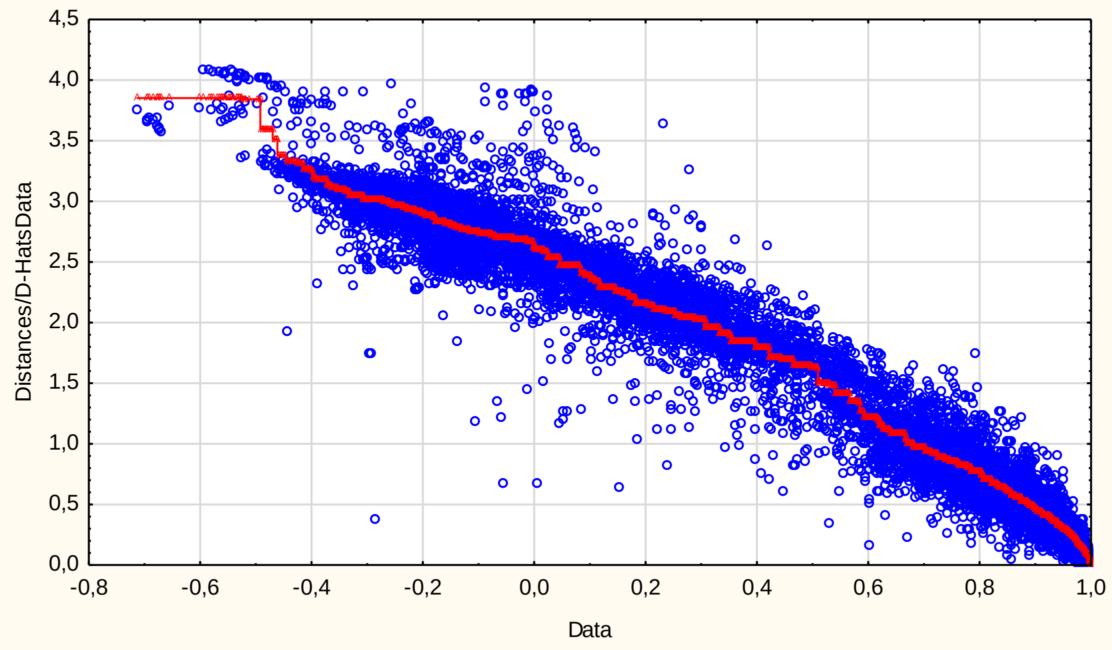

Fig. 9: Shepard diagram
Source: authors, processed in software Statistica

Based on the results of the Kruskal's criterion, which reached the value of 0.08 and the course of the Shepard diagram, it can be stated that the constructed model has a good predictive value.

Summary

The use of multidimensional statistical methods is of great use in assessing the state of the industry and the businesses active within. The paper analyzes the heat management industry and the position of companies within the industry. The applied methods facilitated and accelerated the processing of large amounts of data, made it possible to reduce the number of data dimensions, and thus created a precondition for the use of other important analytical procedures. Thanks to the methods used we were able to display data in simpler and clearer way. The selected group of variables can be replaced by three factors that capture information about the financial situation of a given sample of companies in great detail. These are factors that inform us about the company's profitability, liquidity and capital structure – the most important factors in determining symptoms of bankruptcy. By targeting them, it is possible to improve the financial position of companies and prevent them from going bankrupt. The MDS method makes it possible to identify clusters of companies that are not distant from each other and have similar characteristics. Based on the above, it is possible to identify a group of companies that is expected to go bankrupt. Finally, it should be noted that the results of these methods should be verified by less subjective methods, e.g. discriminant analysis, logistic regression, Data Envelopment Analysis or neural networks.

Súhrn

Používanie viacrozmerných štatistických metód poskytuje značnú pomoc pri hodnotení stavu priemyselného odvetvia a podnikania v ňom. V predmetnom príspevku bolo analyzované odvetvie tepelného hospodárstva a pozícia podnikov v rámci daného odvetvia. Aplikované metódy uľahčili a urýchlili spracovanie veľkého množstva údajov, umožnili zníženie počtu rozmerov údajov, a tým vytvorili predpoklad pre uplatnenie ďalších významných analytických postupov. Prínosom aplikovaných metód je skutočnosť, že grafické zobrazenie predstavuje jednoduchšiu interpretáciu dosiahnutých výsledkov a je prehľadné. Vybranú skupinu premenných je možné nahradiť troma faktormi, ktoré dostatočne zachytávajú informácie

o finančnej situácii danej vzorky podnikov. Ide o faktory, ktoré zachytávajú informácie o rentabilite, likvidite a kapitálovej štruktúre podniku, ktoré patria k symptómom bankrotu podniku. Ich cieľovým riadením je možné zlepšiť finančnú pozíciu podnikov a zvrátiť predpoklad ich bankrotu. Metóda MDS umožňuje identifikovať zhluky podnikov, ktoré nie sú od seba vzdialené a vykazujú podobné vlastnosti. Na základe uvedeného je možné určiť skupinu podnikov, ktorá má predpoklad bankrotu. Na záver je potrebné poznamenať, že výsledky týchto metód je vhodné verifikovať aplikáciou menej subjektívnych metód, a to napr. diskriminačnou analýzou, logistickou regresiou, metódou Data Envelopment Analysis alebo neurónovými sieťami.

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